

## Noncompartmental vs. Compartmental Approaches to Pharmacokinetic Data Analysis

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## Questions To Be Asked

- Pharmacokinetics
  - What the body does to the drug
- Pharmacodynamics
  - What the drug does to the body
- Disease progression
  - Measurable therapeutic effect
- Variability
  - Sources of error and biological variation

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## Pharmacokinetics / Pharmacodynamics



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|------------------------------------|------------------------------------|
| ➤ Pharmacokinetics                 | ➤ Pharmacodynamics                 |
| ➤ “What the body does to the drug” | ➤ “What the drug does to the body” |
| ➤ Fairly well known                | ➤ Largely unknown                  |
| ➤ Useful to get to the PD          | ➤ Has clinical relevance           |

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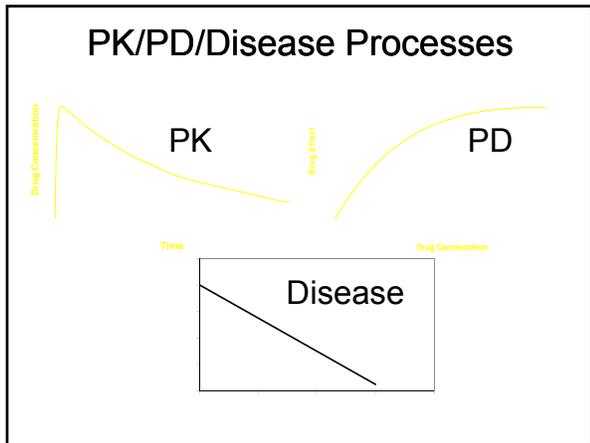
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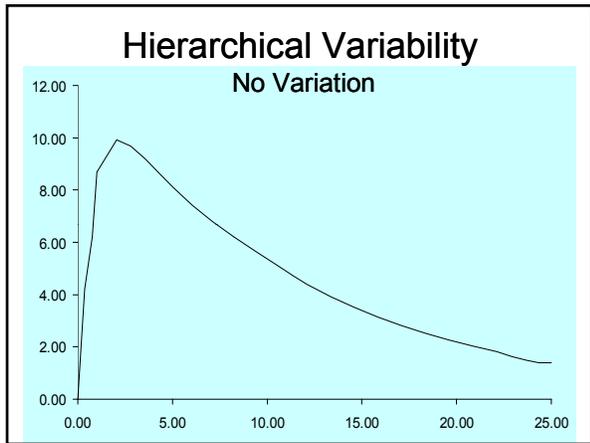
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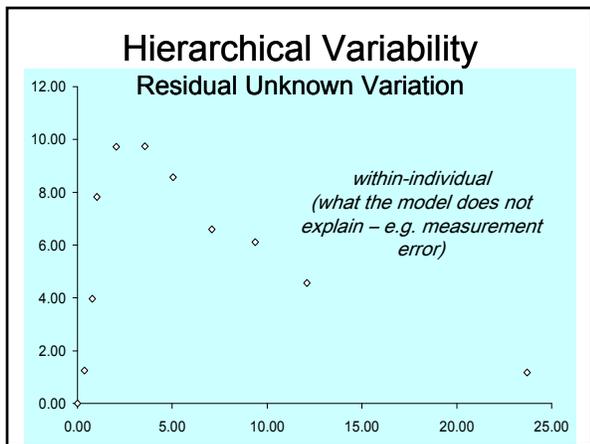
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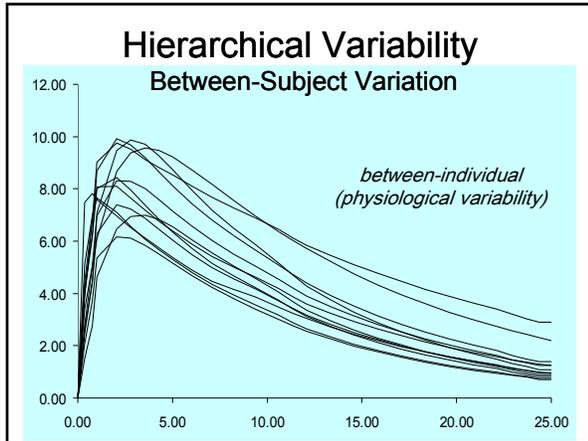
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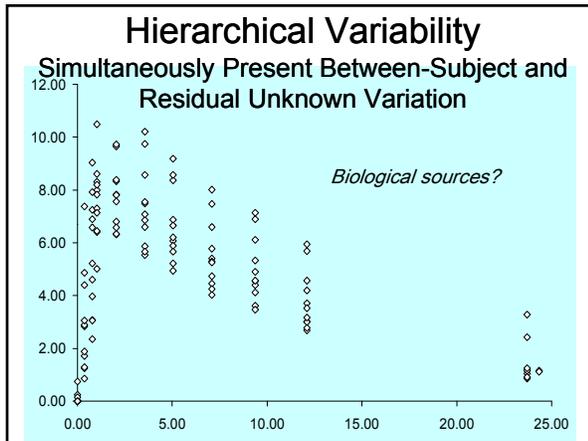
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### Pharmacokinetic Parameters

- Definition of pharmacokinetic parameters
  - Descriptive or observational
  - Quantitative (requiring a formula and a means to estimate using the formula)
- Formulas for the pharmacokinetic parameters
- Methods to estimate the parameters from the formulas using measured data

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## Models For Estimation

Noncompartmental  
Compartmental

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## Goals Of This Lecture

- Description of the parameters of interest
- Underlying assumptions of noncompartmental and compartmental models
- Parameter estimation methods
- What to expect from the analysis

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## Goals Of This Lecture

- What this lecture is about
  - What are the assumptions, and how can these affect the conclusions
  - Make an intelligent choice of methods depending upon what information is required from the data
- What this lecture is not about
  - To conclude that one method is “better” than another

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## A Drug In The Body: Constantly Undergoing Change

- Absorption
  - Transport in the circulation
  - Transport across membranes
  - Biochemical transformation
  - Elimination
- ADME
- Absorption, Distribution, Metabolism, Excretion

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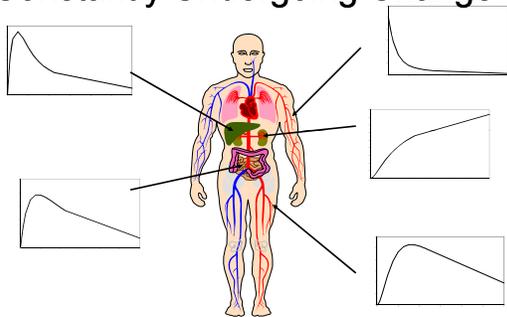
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## A Drug In The Body: Constantly Undergoing Change



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## Kinetics And Pharmacokinetics

- Kinetics
  - The temporal and spatial distribution of a substance in a system.
- Pharmacokinetics
  - The temporal and spatial distribution of a drug (or drugs) in a system.

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## Definition Of Kinetics: Consequences

➤ Spatial: *Where* in the system

- Spatial coordinates
- Key variables: (x, y, z)



➤ Temporal: *When* in the system

- Temporal coordinates
- Key variable: t

$$\frac{\partial c(x, y, z, t)}{\partial x}, \frac{\partial c(x, y, z, t)}{\partial y}, \frac{\partial c(x, y, z, t)}{\partial z}, \frac{\partial c(x, y, z, t)}{\partial t}$$

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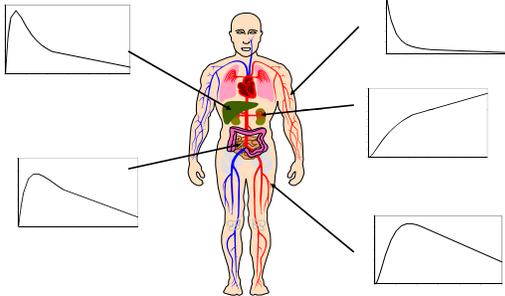
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## A Drug In The Body: Constantly Undergoing Change




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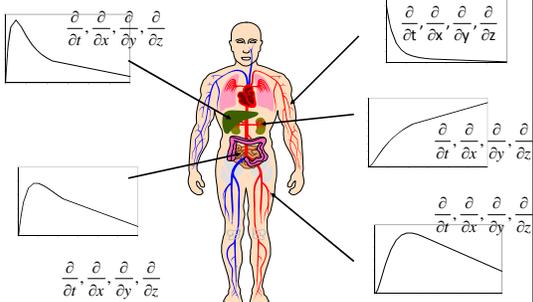
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## A Drug In The Body: Constantly Undergoing Change




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## Spatially Distributed Models

- Spatially realistic models:
  - Require a knowledge of physical chemistry, irreversible thermodynamics and circulatory dynamics.
  - Are difficult to solve.
  - It is difficult to design an experiment to estimate their parameter values.
- While desirable, normally not practical.
- Question: What can one do?

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## Resolving The Problem

- Reducing the system to a finite number of components
- Lumping processes together based upon time, location or a combination of the two
- Space is not taken directly into account: rather, spatial heterogeneity is modeled through changes that occur in time

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## Lumped Parameter Models

- Models which make the system discrete through a lumping process thus eliminating the need to deal with partial differential equations.
- Classes of such models:
  - Noncompartmental models
    - Based on algebraic equations
  - Compartmental models
    - Based on linear or nonlinear differential equations

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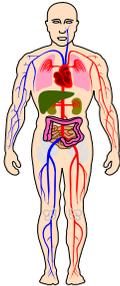
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## Probing The System

➤ **Accessible pools:** These are system spaces that are available to the experimentalist for test input and/or measurement.

➤ **Nonaccessible pools:** These are spaces comprising the rest of the system which are not available for test input and/or measurement.



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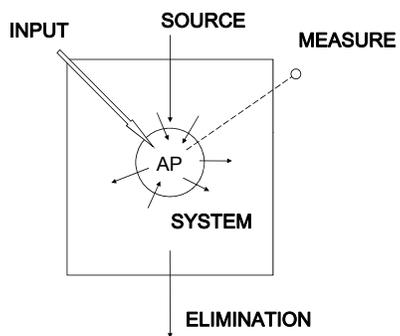
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## Focus On The Accessible Pool



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## Characteristics Of The Accessible Pool

Kinetically Homogeneous  
Instantaneously Well-mixed

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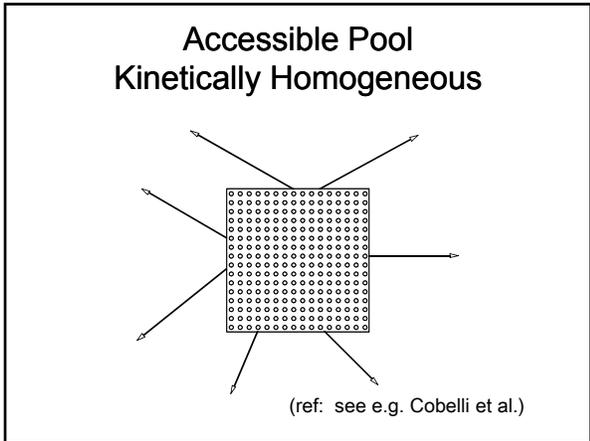
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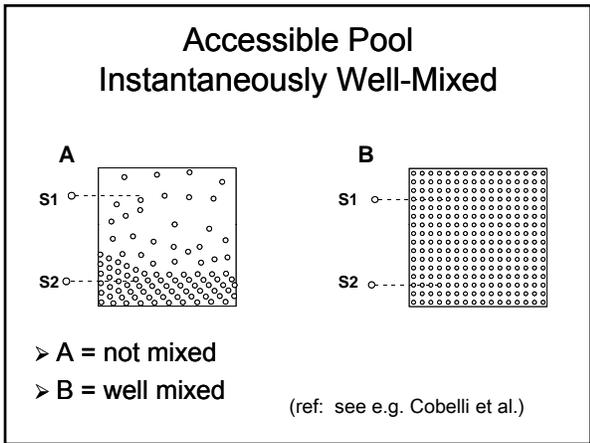
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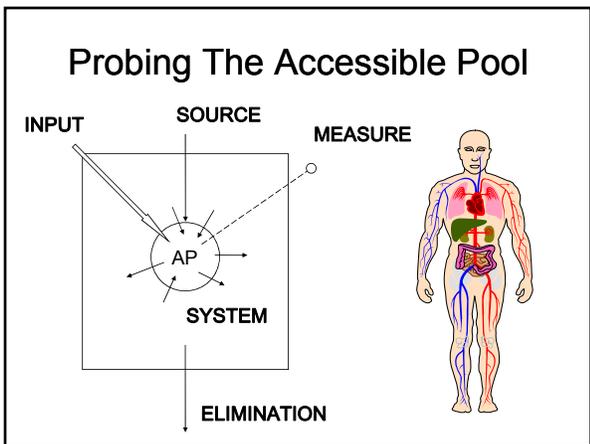
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## The Pharmacokinetic Parameters

- Which pharmacokinetic parameters can we estimate based on measurements in the accessible pool?
- Estimation requires a model
  - Conceptualization of how the system works
- Depending on assumptions:
  - Noncompartmental approaches
  - Compartmental approaches

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## Accessible Pool & System Assumptions → Information

- Accessible pool
  - Initial volume of distribution
  - Clearance rate
  - Elimination rate constant
  - Mean residence time
- System
  - Equivalent volume of distribution
  - System mean residence time
  - Bioavailability
  - Absorption rate constant

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## Compartmental and Noncompartmental Analysis

The only difference between the two methods is in how the nonaccessible portion of the system is described

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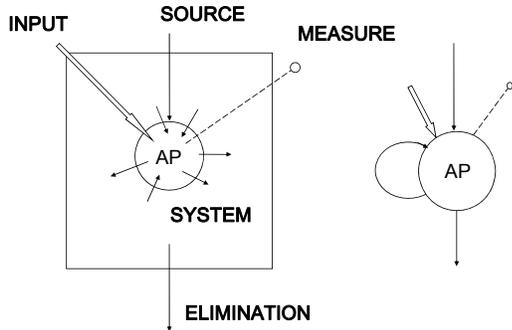
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# The Noncompartmental Model



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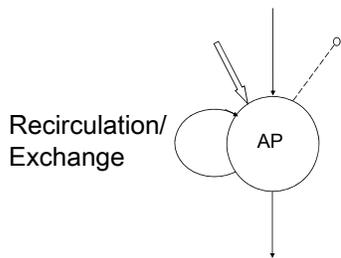
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## Recirculation-exchange Assumptions



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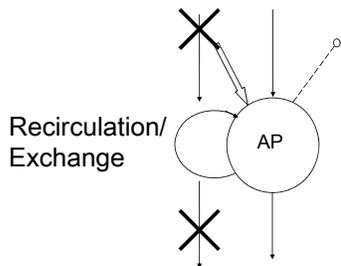
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## Recirculation-exchange Assumptions



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### Single Accessible Pool Noncompartmental Model

- Parameters (IV bolus and infusion)
  - Mean residence time
  - Clearance rate
  - Volume of distribution
- Estimating the parameters from data
- Additional assumption:
  - Constancy of kinetic distribution parameters

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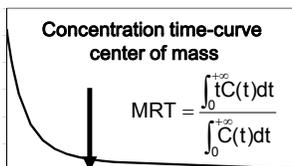
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### Mean Residence Time

- The average time that a molecule of drug spends in the system




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### Areas Under The Curve

- AUMC
  - Area Under the Moment Curve
- AUC
  - Area Under the Curve
- MRT
  - "Normalized" AUMC (units = time)

$$MRT = \frac{\int_0^{+\infty} tC(t)dt}{\int_0^{+\infty} C(t)dt} = \frac{AUMC}{AUC}$$

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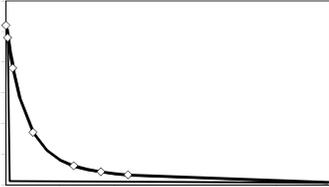
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## What Is Needed For MRT?

- Estimates for AUC and AUMC.




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## What Is Needed For MRT?

- Estimates for AUC and AUMC.

$$AUC = \int_0^{\infty} C(t) dt = \int_0^{t_1} C(t) dt + \int_{t_1}^{t_n} C(t) dt + \int_{t_n}^{\infty} C(t) dt$$

$$AUMC = \int_0^{\infty} t \cdot C(t) dt = \int_0^{t_1} t \cdot C(t) dt + \int_{t_1}^{t_n} t \cdot C(t) dt + \int_{t_n}^{\infty} t \cdot C(t) dt$$

- They require extrapolations beyond the time frame of the experiment
- Thus, this method is not model independent as often claimed.

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## Estimating AUC And AUMC Using Sums Of Exponentials

$$AUC = \int_0^{\infty} C(t) dt = \int_0^{t_1} C(t) dt + \int_{t_1}^{t_n} C(t) dt + \int_{t_n}^{\infty} C(t) dt$$

$$AUMC = \int_0^{\infty} t \cdot C(t) dt = \int_0^{t_1} t \cdot C(t) dt + \int_{t_1}^{t_n} t \cdot C(t) dt + \int_{t_n}^{\infty} t \cdot C(t) dt$$

$$C(t) = A_1 e^{-\lambda_1 t} + \dots + A_n e^{-\lambda_n t}$$

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### Bolus IV Injection

Formulas can be extended to other administration modes

$$AUC = \int_0^{\infty} C(t)dt = \frac{A_1}{\lambda_1} + \dots + \frac{A_n}{\lambda_n}$$

$$AUMC = \int_0^{\infty} t \cdot C(t)dt = \frac{A_1}{\lambda_1^2} + \dots + \frac{A_n}{\lambda_n^2}$$

$$C(0) = A_1 + \dots + A_n$$

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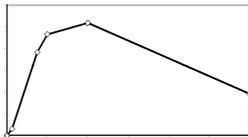
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### Estimating AUC And AUMC Using Other Methods

- Trapezoidal
- Log-trapezoidal
- Combinations
- Other
- Role of extrapolation




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### The Integrals

- These other methods provide formulas for the integrals between  $t_1$  and  $t_n$  leaving it up to the researcher to extrapolate to time zero and time infinity.

$$AUC = \int_0^{t_1} C(t)dt + \int_{t_1}^{t_n} C(t)dt + \int_{t_n}^{\infty} C(t)dt$$

$$AUMC = \int_0^{t_1} t \cdot C(t)dt + \int_{t_1}^{t_n} t \cdot C(t)dt + \int_{t_n}^{\infty} t \cdot C(t)dt$$

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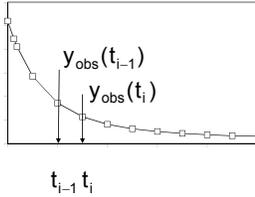
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### Trapezoidal Rule

➤ For every time  $t_i$ ,  $i = 1, \dots, n$

$$AUC_{i-1}^i = \frac{1}{2} [y_{\text{obs}}(t_i) + y_{\text{obs}}(t_{i-1})] (t_i - t_{i-1})$$

$$AUMC_{i-1}^i = \frac{1}{2} [t_i \cdot y_{\text{obs}}(t_i) + t_{i-1} \cdot y_{\text{obs}}(t_{i-1})] (t_i - t_{i-1})$$




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### Log-trapezoidal Rule

➤ For every time  $t_i$ ,  $i = 1, \dots, n$

$$AUC_{i-1}^i = \frac{1}{\ln\left(\frac{y_{\text{obs}}(t_i)}{y_{\text{obs}}(t_{i-1})}\right)} [y_{\text{obs}}(t_i) + y_{\text{obs}}(t_{i-1})] (t_i - t_{i-1})$$

$$AUMC_{i-1}^i = \frac{1}{\ln\left(\frac{y_{\text{obs}}(t_i)}{y_{\text{obs}}(t_{i-1})}\right)} [t_i \cdot y_{\text{obs}}(t_i) + t_{i-1} \cdot y_{\text{obs}}(t_{i-1})] (t_i - t_{i-1})$$

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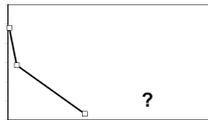
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### Trapezoidal Rule Potential Pitfalls



- As the number of samples decreases, the interpolation may not be accurate (depends on the shape of the curve)
- Extrapolation from last measurement necessary

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## Extrapolating From $t_n$ To Infinity

- Terminal decay is assumed to be a monoexponential
- The corresponding exponent is often called  $\lambda_z$ .
- Half-life of terminal decay can be calculated:

$$t_{z/1/2} = \ln(2) / \lambda_z$$

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## Extrapolating From $t_n$ To Infinity

From last data point:

$$AUC_{\text{extrap-dat}} = \int_{t_n}^{\infty} C(t) dt = \frac{y_{\text{obs}}(t_n)}{\lambda_z}$$

$$AUMC_{\text{extrap-dat}} = \int_{t_n}^{\infty} t \cdot C(t) dt = \frac{t_n \cdot y_{\text{obs}}(t_n)}{\lambda_z} + \frac{y_{\text{obs}}(t_n)}{\lambda_z^2}$$

From last calculated value:

$$AUC_{\text{extrap-calc}} = \int_{t_n}^{\infty} C(t) dt = \frac{A_z e^{-\lambda_z t_n}}{\lambda_z}$$

$$AUMC_{\text{extrap-calc}} = \int_{t_n}^{\infty} t \cdot C(t) dt = \frac{t_n \cdot A_z e^{-\lambda_z t_n}}{\lambda_z} + \frac{A_z e^{-\lambda_z t_n}}{\lambda_z^2}$$

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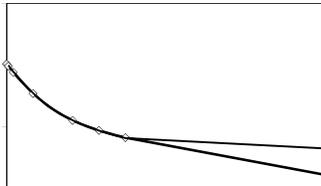
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## Extrapolating From $t_n$ To Infinity

- Extrapolating function crucial




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## Estimating The Integrals

- To estimate the integrals, one sums up the individual components.

$$AUC = \int_0^{\infty} C(t)dt = \int_0^{t_1} C(t)dt + \int_{t_1}^{t_n} C(t)dt + \int_{t_n}^{\infty} C(t)dt$$

$$AUMC = \int_0^{\infty} t \cdot C(t)dt = \int_0^{t_1} t \cdot C(t)dt + \int_{t_1}^{t_n} t \cdot C(t)dt + \int_{t_n}^{\infty} t \cdot C(t)dt$$

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## Advantages Of Using Function Extrapolation (Exponentials)

- Extrapolation is automatically done as part of the data fitting
- Statistical information for all parameters (e.g. their standard errors) calculated
- There is a natural connection with the solution of linear, constant coefficient compartmental models
- Software is available

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## Clearance Rate

- The volume of blood cleared per unit time, relative to the drug

$$CL = \frac{\text{Elimination rate}}{\text{Concentration in blood}}$$

- It can be shown that

$$CL = \frac{\text{DrugDose}}{AUC}$$

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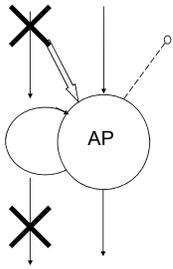
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## Remember Our Assumptions



- > If these are not verified the estimates will be incorrect
- > In addition, this approach cannot straightforwardly handle nonlinearities in the data (time-varying rates, saturation processes, etc.)

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## The Compartmental Model

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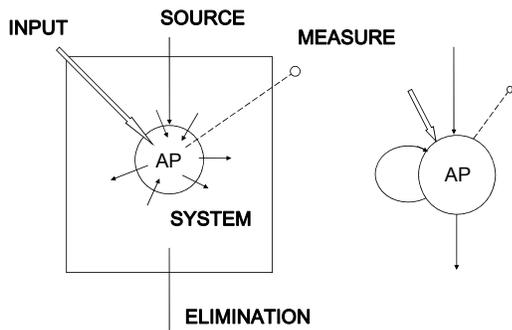
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## Single Accessible Pool



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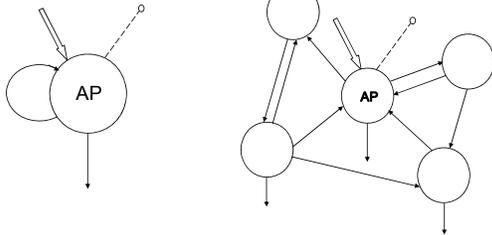
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## Single Accessible Pool Models

- Noncompartmental
- Compartmental



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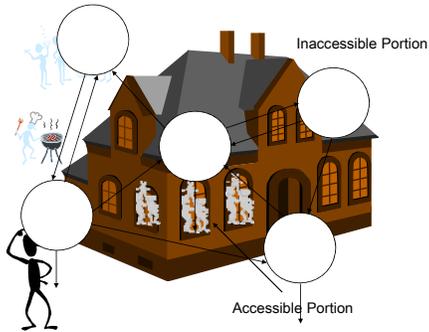
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## A Model Of The System



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## Compartmental Model

- **Compartment**
  - Instantaneously well-mixed
  - Kinetically homogeneous
- **Compartmental model**
  - Finite number of compartments
  - Specifically connected
  - Specific input and output

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## Kinetics And The Compartmental Model

### ➤ Time and space

$$\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}, \frac{\partial}{\partial t}$$

→  $X(x, y, z, t)$

$$\rightarrow \frac{\partial X(x, y, z, t)}{\partial x}, \frac{\partial X(x, y, z, t)}{\partial y}, \frac{\partial X(x, y, z, t)}{\partial z}, \frac{\partial X(x, y, z, t)}{\partial t}$$

### ➤ Time

$$\frac{d}{dt} \rightarrow X(t) \rightarrow \frac{dX(t)}{dt}$$

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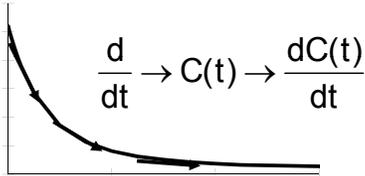
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## Demystifying Differential Equations

➤ It is all about modeling *rates of change*, i.e. *slopes*, or *derivatives*:



➤ Rates of change may be constant or not

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## Ingredients Of Model Building

### ➤ Model of the system

- Independent of experiment design
- Principal components of the biological system

### ➤ Experimental design

- Two parts:
  - Input function (dose, shape, protocol)
  - Measurement function (sampling, location)

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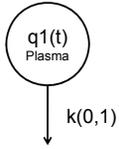
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## Single Compartment Model



$$\frac{dq_1(t)}{dt} = -k(0,1)q_1(t)$$

- > The *rate of change* of the amount in the compartment,  $q_1(t)$ , is equal to what enters the compartment (inputs or initial conditions), minus what leaves the compartment, a quantity proportional to  $q_1(t)$
- >  $k(0,1)$  is a *rate constant*

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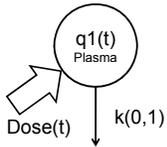
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## Experiment Design Modeling Input Sites



$$\frac{dq_1(t)}{dt} = -k(0,1)q_1(t) + \text{Dose}(t)$$

- > The *rate of change* of the amount in the compartment,  $q_1(t)$ , is equal to what enters the compartment (Dose), minus what leaves the compartment, a quantity proportional to  $q_1(t)$
- >  $\text{Dose}(t)$  can be any function of time

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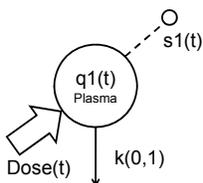
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## Experiment Design Modeling Measurement Sites



$$s_1(t) = \frac{q_1(t)}{V}$$

- > The measurement (sample)  $s_1$  does not subtract mass or perturb the system
- > The measurement equation  $s_1$  links  $q_1$  with the experiment, thus preserving the units of differential equations and data (e.g.  $q_1$  is mass, the measurement is concentration)  
 $\Rightarrow s_1 = q_1 / V$
- >  $V$  = volume of distribution of compartment 1

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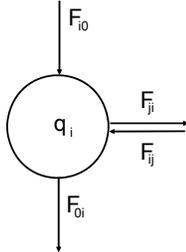
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## Notation

- The fluxes  $F_{ij}$  (from  $j$  to  $i$ ) describe material transport in units of mass per unit time



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## The Compartmental Fluxes ( $F_{ij}$ )

- Describe movement among, into or out of a compartment
- A composite of metabolic activity
  - transport
  - biochemical transformation
  - both
- Similar (compatible) time frame

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## A Proportional Model For The Compartmental Fluxes

- $q$  = compartmental masses
- $p$  = (unknown) system parameters
- $k_{ji}$  = a (nonlinear) function specific to the transfer from  $i$  to  $j$

$$F_{ji}(q, p, t) = k_{ji}(q, p, t) \cdot q_i(t)$$

(ref: see Jacquez and Simon)

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## Nonlinear Kinetics Example

- Remember the one-compartment model:

$$\frac{dq_1(t)}{dt} = -k(0,1)q_1(t) + \text{Dose}(t)$$

$$s_1(t) = \frac{q_1(t)}{V}$$

- What if we had a concentration-dependent drug elimination rate?

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## Nonlinear Kinetics: Michaelis-Menten

- Michaelis-Menten kinetics:

$$\frac{dq_1(t)}{dt} = -\frac{V_m}{K_m + c(t)}q_1(t) + \text{Dose}(t)$$

$$s_1(t) = \frac{q_1(t)}{V}$$

- $V_m$  = maximal metabolic rate
- $K_m$  = Michaelis-Menten constant

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## Linear vs. Nonlinear Kinetics

- If  $K_m \gg c(t)$  then:

$$\frac{dq_1(t)}{dt} \cong -\frac{V_m}{K_m}q_1(t) + \text{Dose}(t)$$

$$s_1(t) = \frac{q_1(t)}{V}$$

- The concentration declines at a rate proportional to it (*first-order kinetics*)
- This is true at *low* concentrations (w.r.t.  $K_m$ )

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## Linear vs. Nonlinear Kinetics

- If  $K_m \ll c(t)$  then:

$$\frac{dq_1(t)}{dt} \cong -\frac{V_m}{c(t)} q_1(t) + D\delta(t) = -V_m \times v_1 + \text{Dose}(t)$$

$$s_1(t) = \frac{q_1(t)}{V}$$

- The concentration declines at a constant rate (*zero-order kinetics*)
- This is true at *high* concentrations (w.r.t.  $K_m$ )

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## Tracking Nonlinearities

- How to find nonlinear behavior?

$$\frac{dq_1(t)}{dt} = -\frac{V_m}{K_m + c(t)} q_1(t) + \text{Dose}(t)$$

$$s_1(t) = \frac{q_1(t)}{V}$$

- **Watch:** Simulated concentration time profile for  $D = 180 \text{ mg}$ ,  $V_m = 20 \text{ mg/L/hr}$ ,  $K_m = 1 \text{ mg/L}$ ,  $v_1 = 5 \text{ L}$

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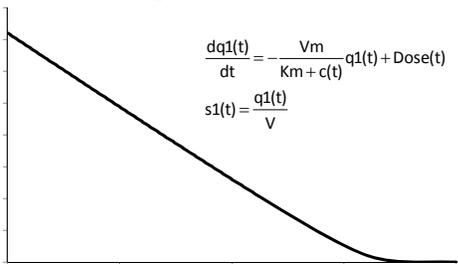
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## Tracking Nonlinearities



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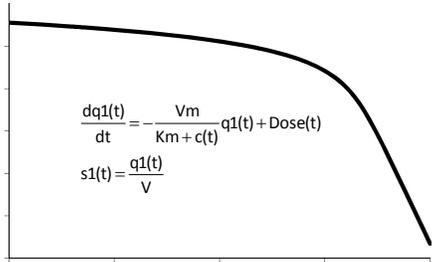
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## Tracking Nonlinearities



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## The Fractional Coefficients ( $k_{ij}$ )

- The fractional coefficients  $k_{ij}$  are called fractional transfer functions
- If  $k_{ij}$  does not depend on the compartmental masses, then the  $k_{ij}$  is called a fractional transfer (or rate) constant.

$$k_{ij}(q, p, t) = k_{ij}$$

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## Compartmental Models And Systems Of Ordinary Differential Equations

- Good mixing
  - permits writing  $q_i(t)$  for the  $i^{\text{th}}$  compartment.
- Kinetic homogeneity
  - permits connecting compartments via the  $k_{ij}$ .

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### The $i^{\text{th}}$ Compartment

Rate of change of  $q_i$

$$\frac{dq_i}{dt} = - \left( \sum_{\substack{j=0 \\ j \neq i}}^n k_{ji}(q, p, t) \right) q_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^n k_{ij}(q, p, t) q_j(t) + F_{i0}$$

Fractional loss of  $q_i$

Fractional input from  $q_j$

Input from "outside" (production rates)

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### Linear, Constant Coefficient Compartmental Models

- All transfer rates  $k_{ij}$  are constant.
  - This facilitates the required computations greatly
- Assume "steady state" conditions.
  - Changes in compartmental mass do not affect the values for the transfer rates

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### The $i^{\text{th}}$ Compartment

Rate of change of  $Q_i$

$$\frac{dQ_i}{dt} = - \left( \sum_{\substack{j=0 \\ j \neq i}}^n k_{ji} \right) Q_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^n k_{ij} Q_j(t) + F_{i0}$$

Fractional loss of  $Q_i$

Fractional input from  $Q_j$

Input from "outside" (production rates)

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## The Compartmental Matrix

$$k_{ii} = - \left( \sum_{j=0, j \neq i}^n k_{ji} \right)$$

$$K = \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1n} \\ k_{21} & k_{22} & \cdots & k_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ k_{n1} & k_{n2} & \cdots & k_{nn} \end{bmatrix}$$

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## Compartmental Model

- A detailed postulation of how one believes a system functions.
- The need to perform the same experiment on the model as one did in the laboratory.

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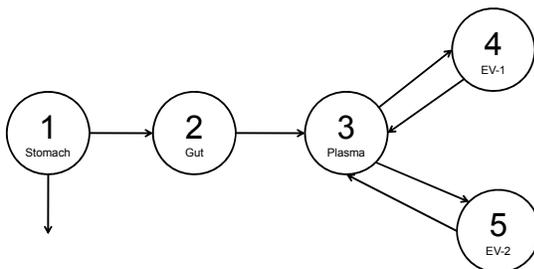
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## Underlying System Model



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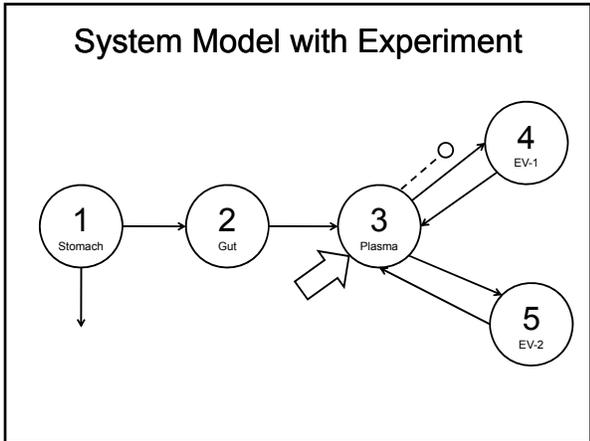
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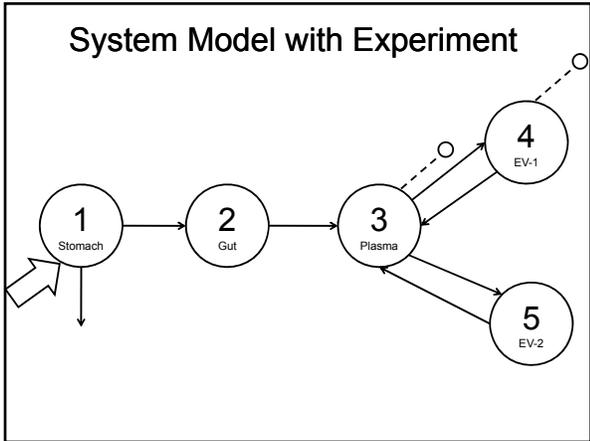
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- ### Experiments
- Need to recreate the laboratory experiment on the model.
  - Need to specify input and measurements
  - Key: UNITS
    - Input usually in mass, or mass/time
    - Measurement usually concentration
      - Mass per unit volume

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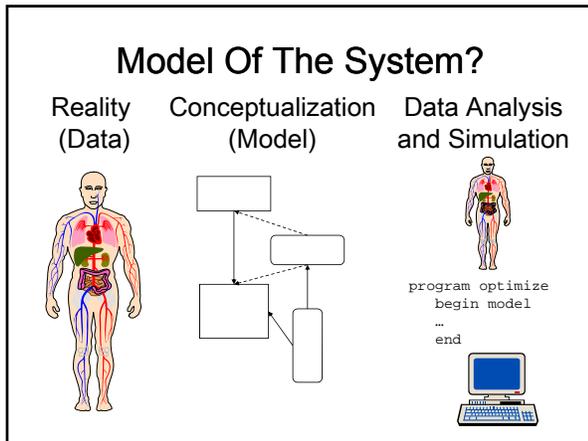
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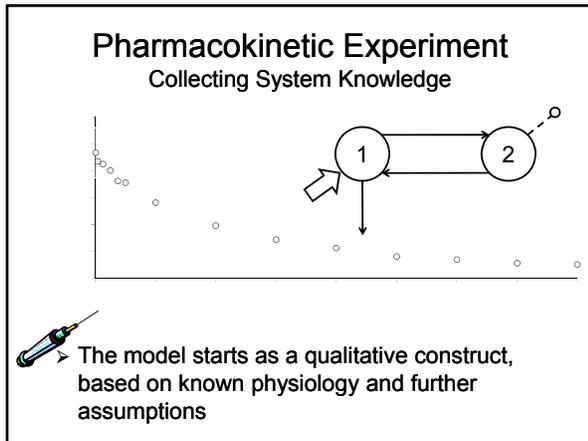
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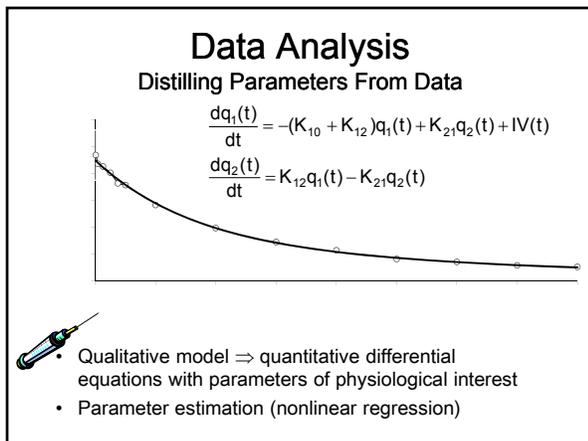
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## Parameter Estimates

- Principles of model building
  - Model definition: structure, error model
  - Model selection: parsimony criteria
  - Estimation methods: maximum likelihood
- Model parameters:  $k_{ij}$  and volumes
- Pharmacokinetic parameters: volumes, clearance, residence times, etc.
- Reparameterization - changing the parameters from  $k_{ij}$  to the PK parameters.

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## Recovering The PK Parameters From Compartmental Models

- Parameters can be based upon
  - The model primary parameters
    - Differential equation parameters
    - Measurement parameters
  - The compartmental matrix
    - Aggregates of model parameters

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## Compartmental Model $\Rightarrow$ Exponential

$$\frac{dq_1(t)}{dt} = -k(0,1)q_1(t) + \text{Dose}\delta(t)$$
$$s_1(t) = \frac{q_1(t)}{V}$$

For a pulse input  $\delta(t)$

$$q_1(t) = \text{Dose} \cdot e^{-k(0,1)t}$$
$$s_1(t) = \frac{q_1(t)}{V} = \frac{\text{Dose}}{V} e^{-k(0,1)t}$$

$$\text{CL} = k(0,1) \times V$$

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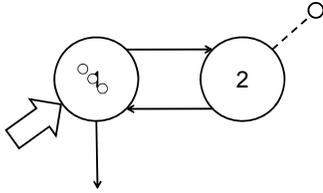
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## Compartmental Residence Times



- Rate constants
- Residence times
- Intercompartmental clearances

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## Parameters Based Upon The Compartmental Matrix

$$K = \begin{bmatrix} k_{11} & k_{12} & \dots & k_{1n} \\ k_{21} & k_{22} & \dots & k_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ k_{n1} & k_{n2} & \dots & k_{nn} \end{bmatrix} \quad \Theta = -K^{-1} = \begin{bmatrix} \vartheta_{11} & \vartheta_{12} & \dots & \vartheta_{1n} \\ \vartheta_{21} & \vartheta_{22} & \dots & \vartheta_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \vartheta_{n1} & \vartheta_{n2} & \dots & \vartheta_{nn} \end{bmatrix}$$

Theta, the negative of the inverse of the compartmental matrix, is called the **mean residence time matrix**.

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## Parameters Based Upon The Compartmental Matrix

### Generalization of Mean Residence Time

$\vartheta_{ij}$  The average time the drug entering compartment  $j$  for the first time spends in compartment  $i$  before leaving the system.

$\frac{\vartheta_{ij}}{\vartheta_{ii}}$ ,  $i \neq j$  The probability that a drug particle in compartment  $j$  will eventually pass through compartment  $i$  before leaving the system.

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## Compartmental Models: Advantages

- Can handle nonlinearities
- Provide hypotheses about system structure
- Can aid in experimental design, for example to design dosing regimens
- Can support translational research

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## Bias That Can Be Introduced By Noncompartmental Analysis

- Not a single sink
  - = Clearance rate
  - ↓ Mean residence time
  - ↓ Volume of distribution
  - ↑ Fractional clearance
- Not a single sink / not a single source
  - ↓ Clearance rate
  - ↓ Mean residence time
  - ↓ Volume of distribution
  - ↑ Fractional clearance

JJ DiStefano III.  
Noncompartmental vs compartmental analysis: some bases for choice.  
Am J. Physiol. 1982;243:R1-R6

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## Nonlinear Pharmacokinetics

- Example: antibody pharmacokinetics
- Often, antibodies exhibit target-mediated disposition, and thus their elimination may occur at sites remote from plasma due to binding and internalization processes
- This is one of many possible biological processes causing nonlinear (capacity-limited) pharmacokinetic behaviors

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## Impact of Noncompartmental Analysis Assumptions

- When drug elimination is influenced by binding to its pharmacological target, the assumptions of noncompartmental analysis may not be met to a varying degree and parameter estimates may be misleading
- Noncompartmental analysis always requires linearity and time invariance, but it can be useful to explore nonlinearities

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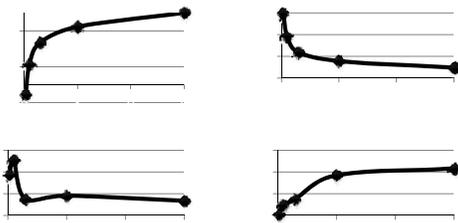
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## Example of Dose Nonlinearities



PK example from Sheremata et al. (1999) as reported in Mager (2006)  
Target-mediated drug disposition and dynamics  
Biochemical Pharmacology 72(2006) 1-10

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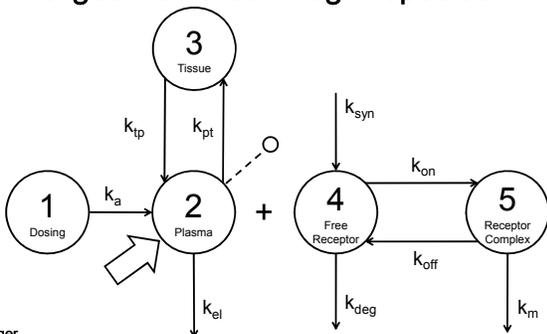
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## Target-Mediated Drug Disposition



Mager  
Target-mediated drug disposition and dynamics  
Biochemical Pharmacology 72(2006) 1-10

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## Take Home Message

- To estimate traditional pharmacokinetic parameters, either model is probably adequate when the sampling schedule is dense, provided all assumptions required for noncompartmental analysis are met
- Sparse sampling schedule and nonlinearities may be an issue for noncompartmental analysis
- Noncompartmental models are not predictive
- Best strategy is probably a blend: but, careful about assumptions!

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